International Journal of Engineering, Science and Mathematics
Vol. 8 Issue 4, April 2019,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

IMPROVING FORECASTS OF GARCH FAMILY MODELS WITH THE ARTIFICIAL NEURAL NETWORKS:AN APPLICATION TO DAILY RETURN VOLATILITY IN MOROCCAN STOCK MARKET.

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Abstract

Stock Market volatility has been extensively studied in finance literature. In this paper, we estimate Moroccan Stock Market return volatility by using Single-State GARCH models and Markov Regime Switching GARCH models. We proposed Back Propagation Neural Network algorithms to improve volatility forecasting of GARCH class models. The BPNN is combined with GARCH in such a way prediction of GARCH models is used as input of our Neural Network. Three volatility estimators are used for this purpose: Absolute return, Parkinson and Garman Klass. The forecasting accuracy of the models is examined using Mean Square Errors (MSE). The results indicate the efficiency of the neural network in enhancing the performance of GARCH models. The findings further clarify the superiority of the marriage of MRS-GARCH and EGARCH with neural network over considered models.

Keywords:

GARCH;
MRSGARCH;
Neural Network;
Volatility.

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1.INTRODUCTION

The study of volatility dynamics has been a prime issue in financial market. In fact, volatility modeling is of paramount importance and its forecast is crucial in asset valuation, risk management and monetary policy design [1]. Several reasons have been advanced as to explain the growing interest in this issue. First, it is a crucial element in assessing market risk. Second, volatility is a key parameter in pricing derivative securities [2]. Third, volatility estimation is essential to building optimal portfolios. Moreover, volatility is a significant factor in determining the bid-ask spread. Indeed, low (high) volatility translates into a narrow (broad) price range. Fifth, financial crises have dramatically increased volatility spillover and contagion among global financial markets. In these regards, the analysis of financial market volatility is more justified by the fact that market shocks can have a huge impact on the real world [3]. Thus, decision-makers base their perception and anticipation of the evolution of the economy by using volatility as a barometer of the strength of the system.

Modelling volatility is often guided by facts characterizing financial time series. Indeed, there are several common features of financial series that are now well documented. The knowledge of such facts may be useful for establishing reliable nonlinear empirical models to forecast volatility. The most popular models in forecasting volatility are GARCH class models. These models are generally broken down into three categories: symmetric volatility models, for which it is the magnitude and not the sign of the shocks that influences the conditional variance (i.e. GARCH (p, q)), asymmetric volatility models, which capture the leverage effects, for instance EGARCH and GJR-GARCH models, and long memory GARCH models (i.e. FIGARCH (p, q)), which accounts for very long-lasting impact of changes in volatility on future movements. That said, some authors suggested taking into account for structural breaks particularly when the sample periods cover financial crises. In that spirit, Lamoureux & Lastrapes[4]suggested using of Markov Regime Switching GARCH models. The mean feature of these relatively new models is that the conditional mean and variance are subject to change across different regimes.

On the other hand, Artificial Neural Network (ANN) is widely used in forecasting financial series. For instance, it is employed in predicting stocks return and constructing optimal portfolios[5], in risk management[6,7]and in financial fraud detection [8]

In that sense, many studies have been conducted to evaluate the accuracy of ANN predictions. T.Datta Chaudhuri and I.Ghosh [9]used nine stock indexes to forecast the Indian stock market volatility. Even that results were satisfying, the selected model was not compared to any other econometric model and does not take advantage from statistical approaches. Hemavati et al.[10]compare the performance of time series based models with ANN which showed tangible enhancement when analyzing Apple stock index volatility. The association of ANN and GARCH models is often involved in forecasting problems. Bildirici & Ersin[11]predict daily return in Istanbul Stock Exchange through different combinations of ANN and GARCH models. In the Moroccan context, Lahmiri[11]attempt to forecast Morrocan index volatility using EGARCH and ANN. He concludes that the trading volume improves the forecast accuracy when used as predictor variable together with EGARCH ouputs. Even though he settles for one GARCH model and did not try different associations.

This paper aims to address the following two questions: 1) Does Markov Regime Switching models provide a superior in-sample fit and better out-sample forecast than the conventional-single-state models? And more importantly: 2) Does the neural networks improve the performance ability of GARCH models in forecasting volatility?

Our research proceeds along the following lines: section 2 provides the methodology and econometric framework. In Section 3, we present the data and the results. Finally, Section 4 concludes this research.

2.RESEARCH METHOD

The methodology used to estimate and forecast volatility in this paper is explained in figure.1 below:

Figure.1. Methodology used for volatility forecasting

1. Data Acquisition.
2. Volatility Estimation: Close, Parkinson, Garman Klass
3. Volatility Modeling: GARCH, EGARCH, GJR-GARCH, MRS-GARCH
4. Improving volatility Forecast: Use of Neural Network
5. Comparing volatility forecast gain: Use of MSE Criterion

2.1.Volatility Estimators

Volatility is a latent parameter but still it can be estimated. The various types of Volatility Estimators used in this research are as follows:

2.1.1. Close method

The simplest and common type of calculation that use only the close price for estimating volatility is the absolute return: $\sigma_t = |r_t| \; (1)$.

2.1.2.Parkinson Method

Parkinson [13] suggests the use of intraday data for estimating volatility. Indeed, he proposed the use of both the highest and lowest prices of each trading day instead of closing prices:

$$\sigma_{P\,(i)} = \sqrt{\frac{(\ln H_i - \ln L_i)^2}{4\ln 2}} \qquad (2)$$

2.1.3.Garman Klass method

Garman Klass method is another extension of Parkinson estimator. The author proposed the following advanced estimator:

$$\sigma_{GK1(i)} = \sqrt{\frac{1}{2}\left[\ln\frac{H_i}{L_i}\right]^2 - 0.39\left[\ln\frac{P_i}{O_i}\right]^2} \qquad (3)$$

2.2. Models

We forecast stock market return volatility using 13 GARCH-model specification/distribution pairs. In the following subsections, we present Autoregressive Conditional Heteroskedasticity Models and Markov Regime Switching Models.

2.2.1.Autoregressive Conditional Heteroskedasticity Models

The GARCH family models consist of substituting the variance constancy hypothesis (homoscedasticity) with a changing conditional variance hypothesis (heteroscedasticity). The GARCH (p, q) makes the conditional volatility a function of p previous square error terms and q past conditional variances. The GARCH (p, q) can be written as follows:

$$r_t = \mu_t + \varepsilon_t = \mu_t + h_t^{1/2}\eta_t, \qquad \eta_t : \; iid\,(0,1), \qquad (4)$$
$$h_t = \alpha_0 + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j h_{t-j}$$

With α_i and β_j being positive parameters, which guarantee that the variance is obviously positive. Taking into account the relationship of recurrence between the conditional variance and its past values, the GARCH model clearly captures the phenomena of persistence. However, Nelson [12]pointed out that the instability of the parameter estimation limits the ability of the model to translate the movements of returns and adequately capture this stylized fact.

Another model employed in this paper is the EGRACH model. Nelson [12]specified the conditional variance in logarithmic form in order to avoid positivity constraints. In addition, this specification considers that a negative shock leads to a high conditional variance with respect to positive shocks. The EGARCH model is formulated as follow:

$$Ln(h_t) = \alpha_0 + \sum_{i=1}^{p}\left[\alpha_i\left\{\left|\frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}} - \sqrt{\frac{\pi}{2}}\right|\right\}\right] + \sum_{j=1}^{q}\left[\gamma_i \frac{\varepsilon_{t-i}}{\sqrt{h_{t-i}}}\right] + \sum_{j=1}^{p}\beta_j L\mathrm{n}(h_{t-j}) \quad (5)$$

An alternative popular GARCH model that takes into account the leverage effect is the GJR-GARCH model (Glosten, Jaanathan, & Runkle, 1993). A GJR-GARCH (1, 1) model is expressed as follows:

$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2 + \beta_1 h_{t-1} \quad (6)$$

$$I_{t-1} = \begin{cases} 1 & \text{when} \quad \varepsilon_{t-1} < 0 \\ 0 & \text{when} \quad \varepsilon_{t-1} \geq 0 \end{cases}$$

The sufficient condition to ensure the positivity of h_t is $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$ and $\alpha + \gamma \geq 0$. The presence of asymmetric effect can be tested by the hypothesis $\gamma \neq 0$. In fact, when $\gamma = 0$, the model collapses to the symmetric GARCH. Notice that, when the shock is negative, the impact of ε_{t-1} is $\alpha_1 + \gamma$; whereas the total effect of a positive shock ε_{t-1} is α_1 .Therefore, we expect γ to be significant and positive as to allow to the model to capture the ‘leverage effect’.

2.2.2.Markov Regime Switching GARCH model

Lamoureux & Lastrapes[4]argues that single-regime GARCH models overestimate the effects of persistence in the presence of structural breaks. Hence, the authors suggested taking into account for structural breaks particularly when the sample periods cover financial crises or sharp and abrupt movements in volatility. In this paper, we adopt the new Markov switching model developed by(16). The general Markov switching GARCH specification can then be expressed as below:

$$y_t \mid (S_t = k, \mathrm{I}_{t-1}) : \quad D\left(0, h_{k,t}, \zeta_k\right) \quad (7)$$

Where $D\left(0, h_{k,t}, \zeta_k\right)$ is a continuous distribution. Notice that y_t denoted as log returns of a financial asset. It has a zero mean, not serially correlated and has a time-varying variance h_{kt}. ζ_k denotes a vector of additional shape parameters. In addition, the variable S_t referring to the state variable characterizes the Markov-Switching Model. Further, Haas et al.[15]state that conditional variance of y_t follows a GARCH-type model. Hence, conditionally on regime $S_t = K$, h_{kt} can be expressed as a function of past returns and variances, and a vector θ_k of additional regime-dependent of parameters.

$$h_{kt} = h(y_{t-1}, h_{k,t-1}, \theta_k) \quad (8)$$

Depending on the form of h (.), we obtain different specifications. For example, the following k separate single-regime conditional variance processes.

$$h_{1,t} = \alpha_{1,0} + \alpha_{1,1} y_{t-1}^2 + \beta_1 h_{1,t-1} \quad (9)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$h_{k,t} = \alpha_{k,0} + \alpha_{k,1} y_{t-1}^2 + \beta_k h_{1,t-1}$$

Moreover, we assume that S_t denoting the state variable evolves to a first order Markov chain, with $K * K$ transition probability matrix P. This matrix dictates the switches between regimes.

$$P = \begin{pmatrix} P_{1,1} & \cdots\cdots & P_{1,k} \\ . & & . \\ . & . & . \\ P_{K,1} & \cdots\cdots & P_{k,k} \end{pmatrix}$$

Where $P_{i,j} = \Pr\ (S_t = j \mid S_{t-1} = i)$ denotes the probability of a transition from state i to state j in the following period

2.3.Conditional distribution

To estimate the parameters of these models, the innovations Z_t are assumed to be following a conditional distribution:

2.3.1. Gaussian Distribution

The normal distribution is the most widely used when estimating GARCH models. In order to estimate the conditional mean, conditional variance and density function, one needs to maximize the following log-likelihood function:

$$L_T = -\frac{1}{2}\sum_{t=1}^{T}[\ln(2\pi) + \ln(\sigma_t^2) + Z_t^2] \qquad (10)$$

2.3.2. Student's Distribution

Taking into consideration that most financial time series have fat tails, one may use the student distribution. For this distribution, the log-likelihood is:

$$L_T = \sum_{t=1}^{T}\ln[\Gamma(\frac{v+1}{2})] - \ln[\Gamma(\frac{v}{2})] - 0.5\ln[\pi(v-2)] - 0.5\sum_{t=1}^{T}[\ln\sigma_t^2 +$$

$$(1+v)\ln(1+\frac{z_t^2}{v-2})] \qquad (11)$$

Where v is the degree of freedom and $\Gamma(.)$ denotes the gamma function. Notice that the student distribution corresponds to a normal distribution when: $v = \infty$.

2.3.3. Generalized Error Distribution

Nelson (1991) proposed the GED as an alternative to student distribution, when estimating EGARCH to ensure the property of stationary. The log-likelihood for the GED is defined as

$$L_{GED} = \sum_{t=1}^{T}[\ln(\frac{v}{\lambda_v} - 0.5\left|\frac{Z_v}{\lambda_v}\right|^v) - (1+v^{-1})\ln(2) - \ln\Gamma(\frac{1}{v}) - 0.5\ln\sigma_t^2] \ (12)$$ where. $$\lambda_v = \sqrt{\frac{\Gamma(\frac{1}{v}2^{\frac{-2}{v}})}{\Gamma(\frac{3}{v})}}$$

2.4. Neural network

Artificial Neural Network is a class of the machine learning algorithms mimicking the human brain behavior. It is used either for classification patterns or prediction of numerical values. An artificial neural network is composed of three categories of layers figure.2:input layer (one layer), hidden layers (one or more) and the output layer. Each layer hosts a number of neurons (brain cells) interconnected to adjacent layers neurons' through numeric values weights (synapses for biological neural network). The output is the result of combining all neurons values associated with weights and activation function. The main idea of ANN is that training the network reduces the gap between outputs (O) and the desired values (Y). There are many training algorithms for ANN, yet the most known is the back propagation algorithm.

Backpropagation Algorithm

In the forward phase, each neuron at the first hidden layer is computed by summing inputs values affected to associated weights.

$$NetHj = \sum_{i}^{m} WijIi + bj \qquad (13)$$

Specific mathematical functions called activation functions, generally nonlinear, is then applied to this combination, since most real-world problems are nonlinear. There is a set of activation functions usedin neural network as heaviside function (14), logistic function (15) and hyperbolic function (16).

$$f(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases} \qquad (14)$$

$$\sigma(x) = \frac{1}{1+e^{-x}} \qquad (15)$$

$$\tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad (16)$$

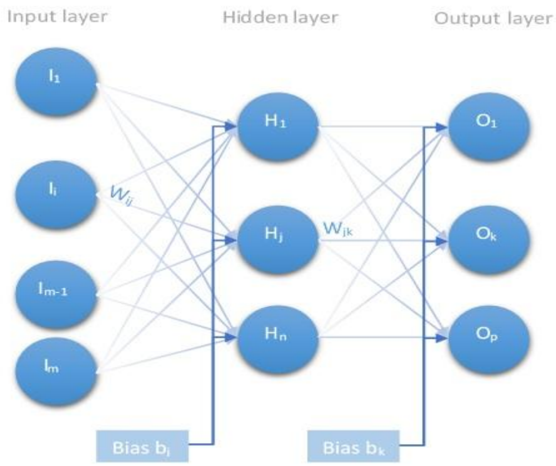

Fig2: Multi-layer back propagation neural network structure

If we consider the hyperbolic function, the value of the neuron j of the first hidden layer is then:

$$Hj = \tanh(\sum_{i}^{m} WijIi + bj) \qquad (17)$$

For the backward propagation, the quadratic cost Cis calculated and weights are updated in such a way C is reduced. The partial derivatives $\frac{\partial C}{\partial w}$ and $\frac{\partial C}{\partial b}$ estimate respectively the behavior of the cost function after small variation of weight w and bias b. Weights and bias are adjusted with the partial derivatives and the cost C is then lowered.

$$C = \frac{1}{2}(Y - O)^2 \qquad (18)$$

$$W = W - \frac{\partial C}{\partial W} \qquad (19)$$

$$b = b - \frac{\partial C}{\partial b} \qquad (20)$$

The forward and backward propagation phases are repeated until the error is getting below a specified measurement. Therefore, the training stage is finished by getting final weights and bias which are used for predicting outputs of new datasets.

Inputs of our designed BNN are historic volatility of the five prior working days predicted by one of the GARCH models (GARCH, MRSGARCH, EGARCH and GJR-GARCH). For example, for the day d inputs are: Vp(d-1), Vp(d-2), Vp(d-3), Vp(d-4) and Vp(d-5). The expected output is the volatility estimated by one of volatility proxies (Absolute return, Parkinson, GK). Hence, the input layer counts five neurons, the output layer has only one neuron and the hidden layer is limited to five neurons. 90% of available data are used for the training and the rest is used for test. Therefore, we have 3313 training examples and 368 test records.

2.5.Comparing performance forecast

We consider volatility forecast comparisons based on expected loss or distance to the true conditional variance. We use the following Criterion:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}\left(\hat{\sigma}_{t+1} - \hat{h}_{t+1|t}^{1/2}\right)^2 \qquad (21)$$

3. DATA AND RESULTS

3.1. Data

The data used in this study consists of daily closing price series for the market index of Casablanca Stock Exchange (MASI) for the period from January 2, 2003 to October 13, 2017. We break down the data into two subsamples. The training set contains the prices from January 2, 2003 to April 22, 2016. The test set contains the prices from April 25, 2016 to October 13, 2017. This out-of-sample set is used for forecasting evaluation. The statistics of daily returns calculated from MASI Index is given in Table 1. In the following, we estimate 13 GARCH-model specification/distribution pairs.

3.2.Volatility modeling

3.2.1 GARCH (1, 1)

According to the results displayed in table 2, all the parameters of the variance equation in GARCH (1, 1) model are significantly different from zero at any traditional level of significance. In fact, the values of the minimum variances represented by α_0 are very small and close to zero, while ARCH effect (α_1), which reflects the impact of past shock on volatility, shows positive values below 0.350 for all distributions. It is also worth noting that the GARCH terms of the models (β_1) seem to be very significantly high. The parameters are superior to 0.60, whatever the distributional nature of innovation is. Furthermore, the volatility is due to the GARCH effect since the GARCH terms are far superior to ARCH effect parameters. This effect is more pronounced when considering the normal distribution. This is a further indication that the market has a long memory and that volatility is more sensitive to its lagged values than to recent market shocks. Moreover, the persistence is expectedly high for all models and under all distributions. However, the results vary depending on the distribution taken in consideration. The longest persistence of shock prevails in the case of the pair 'MASI/Normal'. Further, the condition of stationary is verified ensuring the existence of the unconditional volatility. In fact, the long-run volatilities range between 7.22e-05 and 9.26e-05. It should be highlighted that the mean-reversion, expressed by the so-called half-life is slower especially when we consider a student distribution. Moreover, and based on the criterion of the log-likelihood to choose the best model, we can conclude that, GARCH (1, 1) with a student distribution outperforms all other GARCH models.

3.2.2 EGARCH (1, 1)

Despite the interesting conclusions of the GARCH model, most empirical studies argued that returns are negatively correlated with variations in volatility. In that sense, we will integrate asymmetric evolutions into the dynamics of volatility using EGARCH (1, 1) model with Gaussian, students and GED distribution. As discussed earlier, the model does not place any restrictions on the estimated parameters to ensure non-negativity of the conditional variance. In fact, the constant parameters are negative. Moreover, the regression results show that α_0 α_1 (ARCH effect) and β_1 (GARCH effect) are highly significant at 1%, 5% and 10%. We notice, as in the GARCH (1, 1) model that the GARCH effect is about two times the ARCH effect under all distributions. On the other hand, results show no clear evidence of 'leverage effect'. In fact, the asymmetry parameters γ is negative implying that positive shocks cause short volatility compared to negative shocks of the same magnitude. Concerning the most suitable model for this set, we can retain EGRACH (1, 1) with a student distribution error.

3.2.3 GJR-GARCH (1, 1)

In the case of GJR-GARCH (1, 1) specification, the coefficients α_0, α_1 and β_1 are significant by assuming the three distributions. Analyzing the logarithm of likelihood, we can conclude that the best model is GJR-GARCH (1, 1) with student distribution. Further, the asymmetry parameter γ is positive for all the pairs index /distribution. However, this asymmetric effect is not significant.

3.2.4 MRS-GARCH (1, 1)

To investigate the presence of changes in the regime in volatility, we use the GARCH-Markov Regime Switching models as reported in Table.2. It should be highlighted that the MRS-GARCH model with student distribution can have constant degree of freedom or degrees of freedom that switch between the two regimes. The results unequivocally indicate that all the coefficients in the conditional variance equations arestatistically significant. These results confirm the existence of two states: The lower variance regime that is given by i=1, and the higher variance regime that is noted by i=2. In fact, the first regime unconditional volatilities range between 8.61e-06 and 6.73e-05. In contrast, the second regime volatilities range between 8.46e-05 and 1.61e-03. With respect to persistence, the unconditional probabilities of the Markov chain depend on the distribution of errors. In fact, unconditional probabilities of being in state 2 are very high for Student and GED distributions. Furthermore, the conditional probabilities to shift from state 2 to state one are very slow, which is in line with the fact that most financial series exhibit persistence.

3.3. VolatilityForecasting

3.3.1 In-sample Performance

To assess the performances of competing GARCH models, we use MSE criteria. Note that we use absolute return, Parkinson and Garman Klass as proxies of the true volatility. According to the log-likelihood function, the best model to fit the data is the MRS-GARCH with a student distribution. Over all, it is evident from the results that the MRS-GARCH models outperform the single state GARCH models. These results are expected, as the Markov Switching Models are highly parameterized models (17). Table 3 Confirms this finding since MSE is lowest for MRS-GARCH. Further, table.4 shows that the accuracy of in-sample volatility forecast is enhanced for all GARCH models while combined with BPNN.

3.3.2 Out-of-samplePerformance

Our forecasting Methodology consists of dividing the sample period into a training set covering the first 3318 trading days and the test set for forecast evaluation covering the last 368 days. Indeed, daily forecasts are computed for each day in the forecasting period for one-day horizon. Table.3 reports the out-of-sample evaluation of GARCH models. According to MSE, the best forecasting model for MASI is MRSGARCH-t. Notice that among the single state GARCH models, EGARCH-N provides the best forecast for one-day horizon. Furthermore, the MSE is reduced for NN-GARCH classes compared to GARCH models (Table.4).

4.CONCLUSION

This paper has examined the potential of Neural Network to improve modeling and forecasting the dynamics of volatility. In fact, the main objective of this article is to twofold, first to compare set of a single regime GARCH: GARCH (1, 1), EGARCH (1, 1), GJR-GARCH (1, 1), and Markov Regime-Switching model in terms of modeling volatility and capturing the stylized facts in the Moroccan stock market. Second, to test the ability of Back-propagation Algorithm in improving GARCH volatility forecast.

While the GARCH (1, 1) is used as a benchmark, EGARCH and GJR-GARCH (1, 1) are designed to model asymmetric effects of positive and negative shocks. On the other hand, The MRS-GARCH model is a relatively a new model that allows parameters to switch across different regimes. We also utilize different distributions for the error term, normal and non-normal (Student and GED) errors, with the objective of capturing the heavy tails. Such comparison between these extensive groups of models is conducted by comparing the in sample and out-of-sample forecasting performances. Indeed, out-of sample comparison is carried out by comparing the one-day step volatility forecast. In that sense, we use Mean Square Error (MSE) to select the best model.

The results unequivocally indicate that the MASI index shares all the stylized facts of traditional asset classes. In addition, the single regime GARCH models suggest that GARCH terms are far superior to ARCH effect**.** This is a further indication that the market has a long memory and that volatility is more sensitive to its lagged values than to recent market shocks. Further, we find that all the coefficients of the variance equation for the MS-GARCH model are significant thus suggesting the existence of a second regime. As to forecasting performances, the results reveal that the MRS-GARCH models outperform the single state GARCH models. This entails that the suitable models for in-sample and out-of sample forecasting are MRS-GARCH with a t distribution for MASI. As a result, accounting for structural changes would improve estimating and forecasting the volatility returns of Moroccan Stock Market. From this perspective, the MS-GARCH models seem to open new pathways in modeling volatility in finance, allowing to avoid the shortcomings and inaccuracies of the classical modeling and to accurately presaging the extreme risks in the financial markets. Moreover, the accuracy of in-sample and out of sample volatility forecast is enhanced for all GARCH models while combined with BPNN. These findings could be of particular use to investors and academics interested in the forecasting of daily volatility in the Moroccan context.

Minimum	Median	Mean	Maximum	S.D	Skewness	Kurtosis
-0.0502	0.0004	0.0004	0.0446	0.0077	-0.2954	5.3450

Table 1. Statistics of daily percentage returns, MASI Stock Index

		State	omega	ARCH	GARCH	Asymmetry	nu	Log likelihood	p11	p21	π
GARCH	GARCH-Norm		3.51e-06***	2.17e-01***	7.34e-01***			11932			
	GARCH-t		5.45e-06***	3.17e-01***	6.23e-01***		5.027***	12057			
	GARCH-GED		4.49e-06***	2.63e-01***	6.74e-01***		1.238***	12051			
EGARCH	EGARCH-Norm		-0.619***	0.3611***	0.94***	-0.0191**		11936			
	EGARCH-t		-0.853***	0.43***	0.91***	-0.0199***	4.972***	12057			
	EGARCH-GED		-0.746***	0.39***	0.92***	-0.0189*	1.240***	12053			
GJR-GARCH	GJR-GARCH-Norm		0.000***	0.20***	0.73***	0.0290		11933			
	GJR-GARCH-t		0.000***	0.29***	0.62***	0.0429	5.036***	12058			
	GJR-GARCH-GED		0.000***	0.25***	0.67***	0.0354	1.239***	12052			
MRS-GARCH	MRS-GARCH-Norm	1	5.48e-07***	2.03e-02	9.23e-01***			12067	8.88e-01***		0.66
		2	7.82e-06***	1.51e-01	8.44e-01***					2.19e-01***	0.34
	MRS-GARCH-t2	1	1.00e-07***	5.17e-02***	9.37e-01***		3.995***	12080	7.23e-01***		0.54
		2	8.81e-06***	5.23e-01***	4.23e-01***		9.967***			3.27e-01***	0.46
	MRS-GARCH-t	1	1.00e-07***	2.96e-02***	9.65e-01***		6.448***	12084	4.33e-01***		0.39
		2	5.90e-06***	5.38e-01***	4.39e-01***					3.68e-01***	0.61
	MRS-GARCH-GED	1	1.19e-07***	3.24e-02***	9.66e-01***		1.395***	12082	3.82e-01***		0.36
		2	5.10e-06***	4.90e-01***	4.49e-01***					3.43e-01***	0.64

Table 2. Volatility Estimation using GARCH class models

	GARCH			MRSGARCH				EGARCH			GJR-G	
	NORMAL	STD	GED	NORMAL	STD2	STD	GED	NORMAL	STD	GED	NORMAL	
Abs. eturn	3.09E-5	3.16E-5	3.09E-5	2.99E-5	2.98E-5	3.03E-5	3.01E-5	3.01E-5	3.05E-5	2.99E-5	3.09E-5	3
ARK	1.54E-5	1.63E-5	1.53E-5	1.39E-5	1.45E-5	1.51E-5	1.49E-5	1.42E-5	1.47E-5	1.40E-5	1.54E-5	1
GK	1.62E-5	1.73E-5	1.62E-5	1.44E-5	1.53E-5	1.60E-5	1.58E-5	1.48E-5	1.55E-5	1.46E-5	1.62E-5	1
Ab. eturn	2.54E-5	2.60E-5	2.53E-5	2.54E-5	2.43E-5	2.47E-5	2.46E-5	2.50E-5	2.54E-5	2.49E-5	2.53E-5	2
ARK	1.21E-5	1.29E-5	1.22E-5	1.22E-5	1.11E-5	1.15E-5	1.13E-5	1.15E-5	1.20E-5	1.14E-5	1.20E-5	1
GK	1.15E-5	1.26E-5	1.17E-5	1.17E-5	1.05E-5	1.09E-5	1.07E-5	1.07E-5	1.13E-5	1.07E-5	1.14E-5	1

		NN-GARCH			NN-MRSGARCH				NN-EGARCH		
		NORMAL	STD	GED	NORMAL	STD2	STD	GED	NORMAL	STD	GED
In Sample	Abs. Return	2.83E-05	2.85E-5	2.86E-5	2.84E-5	2.80E-5	2.80E-5	2.80E-5	2.80E-5	2.82E-5	2.85E-5
	PARK	9.55E-06	9.77E-6	9.55E-6	9.40E-6	9.37E-6	9.36E-6	9.32E-6	9.30E-6	9.35E-6	9.43E-6
	GK	7.39E-06	7.60E-6	7.47E-6	7.29E-6	7.32E-6	7.20E-6	7.23E-6	7.38E-6	7.24E-6	7.37E-6
Out Of Sample	Abs. Return	2.29E-05	2.32E-5	2.33E-5	2.30E-5	2.28E-5	2.31E-5	2.30E-5	2.32E-5	2.32E-5	2.31E-5
	PARK	9.51E-06	9.64E-6	9.56E-6	9.55E-6	9.43E-6	9.51E-6	9.46E-6	9.37E-6	9.30E-6	9.30E-6
	GK	7.30E-06	7.39E-6	7.33E-6	7.22E-6	7.16E-6	7.18E-6	7.28E-6	7.09E-6	7.18E-6	7.17E-6

Table 3. MSE of Volatility Estimation using GARCH class models

Table 4. MSE of Volatility Estimation using NN-GARCH class models

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